Section 3.4: Exponential and Logarithmic Equations and Inequalities

- Solving Exponential and Logarithmic Equations
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Solving Exponential and Logarithmic Equations

Solving Exponential Equations:

An equation in which the variable appears in the exponent is called an exponential equation.

To solve such an equation, isolate the exponential expression on one side of the equation and take the logarithm of both sides to solve for the variable.

In solving exponential equations, it is helpful to apply the following property of logarithms which is an immediate consequence of the definition.

\[ \log_a a^c = c \]

Example:

Solve the exponential equation \( 2 + e^{2-5x} = 8 \).

(a) Give exact values for all solutions.
(b) Use a calculator to approximate the solutions found in part (a). Give results to the nearest thousandth.
(c) Rewrite the results in part (a) if necessary so that each logarithm contains a prime number.
Solution:

Part (a):

\[ 2 + e^{2-5x} = 8 \]
\[ e^{2-5x} = 6 \]
\[ \ln(e^{2-5x}) = \ln(6) \quad \text{To isolate the exponential expression, subtract 2 from both sides.} \]
\[ 2 - 5x = \ln(6) \]
\[ 2 - 5x - 2 = \ln(6) - 2 \]
\[ -5x = \ln(6) - 2 \]
\[ x = \frac{-5}{-5} \]
\[ x = \frac{-5}{-5} = 1 \]

Take the logarithm to the base \( e \) of both sides.

\[ \log_a a^b = c \]

Solve for \( x \). Begin by subtracting 2 from both sides.

Part (b):

\[ x = \frac{2 - \ln(6)}{5} \approx 0.042 \]

Part (c):

\[ x = \frac{2 - \ln(6)}{5} = \frac{2 - \ln(2 \cdot 3)}{5} \]
\[ = \frac{2 - [\ln(2) + \ln(3)]}{5} \]
\[ = \frac{2 - \ln(2) - \ln(3)}{5} \]
Solving Logarithmic Equations:

An equation in which the logarithm of the variable occurs is called a logarithmic equation.

To solve such an equation, isolate the logarithmic term on one side of the equation and write the equation in exponential form or raise the base to each side of the equation to solve for the variable.

Example:
Solve the logarithmic equation $\log_4(x) + \log_4(x + 6) = 2$.

Solution:

\[
\begin{align*}
\log_4(x) + \log_4(x + 6) &= 2 \\
\log_4(x(x + 6)) &= 2 & \text{Use the laws of logarithms.} \\
x(x + 6) &= 4^2 & \text{Write in exponential form.} \\
x^2 + 6x &= 16 \\
x^2 + 6x - 16 &= 16 - 16 \\
x^2 + 6x - 16 &= 0 \\
(x + 8)(x - 2) &= 0
\end{align*}
\]

\[
\begin{align*}
x + 8 &= 0 & \text{or} & & x - 2 &= 0 \\
x + 8 - 8 &= 0 - 8 & & x - 2 + 2 &= 0 + 2 \\
x &= -8 & & x &= 2
\end{align*}
\]

\(-8\) is not a solution of the given equation since logarithms are only defined for positive numbers.

\[
\begin{align*}
x &= -8 \\
x &= 2
\end{align*}
\]

The solution is $x = 2$.

Example:
Solve the logarithmic equation $2\log(x) = \log(4) + \log(x + 3)$. 
Solution:

\[ 2 \log(x) = \log(4) + \log(x + 3) \]
\[ \log(x^2) = \log(4(x + 3)) \quad \text{Use the laws of logarithms.} \]
\[ \log(x^2) = \log(4x + 12) \]
\[ x^2 = 4x + 12 \]
\[ x^2 - 4x - 12 = 0 \]
\[ (x + 2)(x - 6) = 0 \]
\[ x + 2 = 0 \quad \text{or} \quad x - 6 = 0 \]
\[ x = -2 \quad x = 6 \]

\(-2\) is not a solution of the given equation since logarithms are only defined for positive numbers.

\[ x = 6 \]

The solution is \( x = 6 \).

Example:

Solve the logarithmic equation \( (\ln(x))^2 = 2\ln(x) \).

Solution:

\[ (\ln(x))^2 = 2\ln(x) \]
\[ (\ln(x))^2 - 2\ln(x) = 0 \quad \text{Subtract 2ln(x) from both sides.} \]
\[ \ln(x)(\ln(x) - 2) = 0 \quad \text{Factor on the LHS.} \]
\[ \ln(x) = 0 \quad \text{or} \quad \ln(x) - 2 = 0 \]
\[ \ln(x) = 0 \quad \ln(x) = 2 \]
\[ x^0 = x \quad e^2 = x \quad \text{Write each equation in exponential form.} \]
\[ 1 = x \quad e^2 = x \]

The solutions are \( x = 1 \) and \( x = e^2 \).
Additional Example 1:

Solve the exponential equation $1 + e^{2x} = 5$.

(a) Give exact values for all solutions. Use natural logarithms if logarithms appear in the answer.

(b) Use a calculator to approximate the solutions found in part (a). Give results to the nearest thousandth.

(c) Rewrite the results in part (a) if necessary so that each logarithm contains a prime number.

Solution:

Part (a):
To solve the equation, isolate the exponential expression on the LHS.

\[
1 + e^{2x} = 5 \\
1 + e^{2x} - 1 = 5 - 1 \\
e^{2x} = 4 \\
\ln(e^{2x}) = \ln(4) \\
2x = \ln(4) \\
\frac{2x}{2} = \frac{\ln(4)}{2} \\
x = \frac{\ln(4)}{2}
\]

Part (b):
Use a calculator to give the result to three decimal places.

\[
x = \frac{\ln(4)}{2} \approx 0.693
\]

Part (c):

\[
x = \frac{\ln(4)}{2} \\
= \frac{\ln(2^2)}{2} \\
= \frac{\ln(2^2)}{2} \\
= \frac{2\ln(2)}{2} \quad \text{Use the laws of logarithms.} \\
= \ln(2)
\]
Additional Example 2:

Solve the exponential equation $6^{-4x} = 15$.

(a) Give exact values for all solutions. Use natural logarithms if logarithms appear in the answer.

(b) Use a calculator to approximate the solutions found in part (a). Give results to the nearest thousandth.

(c) Rewrite the results in part (a) if necessary so that each logarithm contains a prime number.

Solution:

Part (a):

To solve the equation, take the natural logarithm of both sides.

\[
\ln \left(6^{-4x}\right) = \ln(15)
\]

\[-4x \cdot \ln(6) = \ln(15)\]

Use the laws of logarithms.

\[
\frac{-4x \cdot \ln(6)}{\ln(6)} = \frac{\ln(15)}{\ln(6)}
\]

\[-4x = \frac{\ln(15)}{\ln(6)}\]

\[
\left(-\frac{1}{4}\right) \cdot (-4x) = \left(-\frac{1}{4}\right) \left(\frac{\ln(15)}{\ln(6)}\right)
\]

\[x = -\frac{\ln(15)}{4\ln(6)}\]

Part (b):

Use a calculator to give the result to three decimal places.

\[x = -\frac{\ln(15)}{4\ln(6)} \approx -0.378\]

Part (c):

\[x = -\frac{\ln(15)}{4\ln(6)}\]

\[= -\frac{\ln(5 \cdot 3)}{4\ln(2 \cdot 3)}\]

\[= -\frac{\ln(5) + \ln(3)}{4[\ln(2) + \ln(3)]}\]

Use the laws of logarithms.
Additional Example 3:

Solve the exponential equation $e^{2x} - 14e^x + 48 = 0$.

(a) Give exact values for all solutions. Use natural logarithms if logarithms appear in the answer.

(b) Use a calculator to approximate the solutions found in part (a). Give results to the nearest thousandth.

(c) Rewrite the results in part (a) if necessary so that each logarithm contains a prime number.

Solution:

Part (a):

\[
e^{2x} - 14e^x + 48 = 0
\]

\[
(e^x - 6)(e^x - 8) = 0
\]

\[
e^x - 6 = 0 \quad \text{or} \quad e^x - 8 = 0
\]

\[
e^x - 6 + 6 = 0 + 6 \quad e^x - 8 + 8 = 0 + 8
\]

\[
e^x = 6 \quad e^x = 8
\]

\[
\ln(e^x) = \ln(6) \quad \ln(e^x) = \ln(8)
\]

\[
x = \ln(6) \quad x = \ln(8)
\]

The solutions are $x = \ln(6)$ and $x = \ln(8)$.

Part (b):

Use a calculator to give the results to three decimal places.

\[
x = \ln(8) \approx 2.079
\]

\[
x = \ln(6) \approx 1.792
\]

Part (c):

\[
x = \ln(8) = \ln\left(2^3\right) = 3\ln(2)
\]

\[
x = \ln(6) = \ln(2 \cdot 3) = \ln(2) + \ln(3)
\]
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Additional Example 4:
Solve the logarithmic equation $\log_2(x + 1) = 5$.

Solution:

\[
\begin{align*}
\log_2(x + 1) & = 5 \\
x + 1 & = 2^5 \\
x + 1 & = 32 \\
x + 1 - 1 & = 32 - 1 \\
x & = 31
\end{align*}
\]

Additional Example 5:
Solve the logarithmic equation $\log(4x + 3) - 2 = -3$.

Solution:

\[
\begin{align*}
\log(4x + 3) - 2 & = -3 \\
\log(4x + 3) - 2 + 2 & = -3 + 2 \\
\log(4x + 3) & = -1 \\
10^{-1} & = 4x + 3 \\
\frac{1}{10} & = 4x + 3 \\
\frac{1}{10} \cdot \frac{1}{10} & = 10(4x + 3) \\
1 & = 40x + 30 \\
1 - 30 & = 40x + 30 - 30 \\
-29 & = 40x \\
\frac{-29}{40} & = \frac{40x}{40} \\
-\frac{29}{40} & = x
\end{align*}
\]

Additional Example 6:
Solve the logarithmic equation $\log_2(x) + \log_2(x + 3) = 2$. 
Solution:
\[
\log_2(x) + \log_2(x + 3) = 2
\]
\[
\log_2 \left( x(x + 3) \right) = 2
\]
Use the laws of logarithms.
\[
x(x + 3) = 2^2
\]
Write in exponential form.
\[
x^2 + 3x = 4
\]
\[
x^2 + 3x - 4 = 4 - 4
\]
\[
x^2 + 3x - 4 = 0
\]
\[
(x + 4)(x - 1) = 0
\]
\[
x + 4 = 0 \quad \text{or} \quad x - 1 = 0
\]
\[
x + 4 - 4 = 0 - 4 \quad x - 1 + 1 = 0 + 1
\]
\[
x = -4 \quad \quad \quad x = 1
\]

-4 is not a solution of the given equation since logarithms are only defined for positive numbers.

\[
x = -4 \quad \quad \quad x = 1
\]

The solution is \( x = 1 \).

Solving Exponential and Logarithmic Inequalities

Properties of Inequality Involving Exponential Functions:

Let \( f(x) = a^x \) with \( a > 1 \).

For all real numbers \( x \), \( a^x > 0 \). Moreover, the following two properties hold:

1. If \( m < n \), then \( a^m < a^n \).
2. If \( a^m > a^n \), then \( m < n \).

See the graph below.
Properties of Inequality Involving Logarithmic Functions:

Let $f(x) = \log_a(x)$ with $a > 1$.

For $0 < x < 1$, $\log_a(x)$ is negative and for $x > 1$, $\log_a(x)$ is positive. Moreover, for positive numbers $m$ and $n$, the following two properties hold:

1. If $m < n$, then $\log_a(m) < \log_a(n)$.
2. If $\log_a(m) < \log_a(n)$, then $m < n$.

See the graph below.

\[ f(x) = \log_a(x) \ (a > 1) \]
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Example:
Solve the inequality: \(6\left(3 - 0.4^x\right) \leq 3\)

Solution:
\[
6\left(3 - 0.4^x\right) \leq 3
\]
\[
3 - 0.4^x \leq \frac{1}{2}
\]
Divide both sides by 6.
\[
-0.4^x \leq -\frac{5}{2}
\]
Subtract 3 from both sides.
\[
0.4^x \geq \frac{5}{2}
\]
Divide both sides by \(-1\). This reverses the inequality symbol.
\[
\ln\left(0.4^x\right) \geq \ln\left(\frac{5}{2}\right)
\]
Apply properties of inequality for logarithmic functions.
\[
x\ln(0.4) \geq \ln\left(\frac{5}{2}\right)
\]
Use laws of logarithms. \(\log_a\left(A^C\right) = C\log_a(A)\)
\[
x \leq \frac{\ln\left(\frac{5}{2}\right)}{\ln(0.4)}
\]
Divide both sides by \(\ln(0.4)\). This reverses the inequality symbol since \(\ln(0.4) < 0\).
\[
x \leq \frac{\ln(5) - \ln(2)}{\ln(2) - \ln(5)}
\]
Use laws of logarithms. \(\log_a\left(\frac{A}{B}\right) = \log_a(A) - \log_a(B)\)
\[
x \leq -1
\]
Simplify.

The solution set in interval notation is \((-\infty, -1]\).

Example:
Solve the inequality: \(\ln(5 - 3x) \leq 1\)

Solution:
Step 1: We need to determine the domain of the function \(y = \ln(5 - 3x)\).
Note that for \(\ln(5 - 3x)\) to be a real number, we must have \(5 - 3x > 0\). Thus, we must solve the inequality \(5 - 3x > 0\).
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\[ 5 - 3x > 0 \]
\[ -3x > -5 \quad \text{Subtract 5 from both sides.} \]
\[ x < \frac{5}{3} \quad \text{Divide both sides by } -3. \text{ This reverses the inequality symbol.} \]

Step 2: Now, we return to the inequality \( \ln (5 - 3x) \leq 1. \)

\[
\ln (5 - 3x) \leq 1 \\
e^{\ln (5 - 3x)} \leq e^1 \\
5 - 3x \leq e \quad \text{Apply properties of inequality for exponential functions.} \\
-3x \leq e - 5 \quad \text{Simplify. (For } c > 0, \ e^{\ln c} = c) \\
x \geq \frac{5 - e}{3} \quad \text{Subtract 5 from both sides.} \\
\]
\[ x \geq \frac{5 - e}{3} \quad \text{Divide both sides by } -3. \text{ This reverses the inequality symbol.} \]

To determine the solution set, we combine the results from steps 1 and 2 above.

First, we must have \( x \) less than \( \frac{5}{3} \). Secondly, we must also have \( x \) greater than or equal to \( \frac{5 - e}{3} \approx 0.76 \).

Thus, the solution set in interval notation is \( \left[ \frac{5 - e}{3}, \frac{5}{3} \right) \).

Example:

Solve the inequality \( 7^{x+4} > 20. \)

Solution:

\[ 7^{x+4} > 20 \]
\[ \ln \left( 7^{x+4} \right) > \ln(20) \quad \text{Apply properties of inequality for logarithmic functions.} \]
\[ (x + 4) \ln(7) > \ln(20) \quad \text{Use laws of logarithms.} \quad \log_a \left( A^C \right) = C \log_a (A) \]
\[ x + 4 > \frac{\ln(20)}{\ln(7)} \quad \text{Divide both sides by } \ln(7). \]
\[ x > \frac{\ln(20)}{\ln(7)} - 4 \quad \text{Subtract 4 from both sides.} \]
\[ x > \frac{\ln(20) - 4 \ln(7)}{\ln(7)} \]
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The solution set in interval notation is \( \left( \frac{\ln(20) - 4\ln(7)}{\ln(7)}, \infty \right) \).

Additional Example 1:

Solve the following inequalities.

(a) \( 6^x > 0 \)
(b) \( \log_5 x \geq 0 \)
(c) \( 5^x < 0 \)
(d) \( \log_5 x < 0 \)

Solution:

Part (a):

Use the following property of inequality for exponential functions:

For all real numbers \( x \), \( a^x > 0 \).

Thus, for each real number \( x \), we have \( 6^x > 0 \). The solution set in interval notation is \( (-\infty, \infty) \).

Part (b):

Use the following property of inequality for logarithmic functions:

If \( a > 1 \) and \( x \geq 1 \), then \( \log_a x \geq 0 \).

Thus, for \( x \geq 1 \), we have \( \log_5 x \geq 0 \). The solution set in interval notation is \( [1, \infty) \).

Part (c):

Use the following property of inequality for exponential functions:

For all real numbers \( x \), \( a^x > 0 \).

Thus, for each real number \( x \), we must have \( 5^x > 0 \). Thus, the given inequality \( 5^x < 0 \) has no solution. The solution set is \( \emptyset \).

Part (d):

Use the following property of inequality for logarithmic functions:

If \( a > 1 \) and \( 0 < x < 1 \), then \( \log_a x < 0 \).

Thus, for \( 0 < x < 1 \), we have \( \log_5 x < 0 \). The solution set in interval notation is \( (0, 1) \).
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Additional Example 2:
Solve the inequality $e^{2-x} > 12$.

Solution:

$e^{2-x} > 12$

$\ln \left( e^{2-x} \right) > \ln(12)$  
Apply properties of inequality for logarithmic functions.

$2 - x > \ln(12)$  
Simplify. ($\ln(e^x) = x$ for all real numbers $x$)

$-x > \ln(12) - 2$  
Subtract 2 from both sides.

$x < 2 - \ln(12)$  
Multiply both sides by $-1$. This reverses the inequality symbol.

The solution set in interval notation is $(-\infty, 2 - \ln(12))$.

Additional Example 3:
Solve the inequality $\ln(4 - 5x) < 3$.

Solution:

Step 1: We need to determine the domain of the function $y = \ln(4 - 5x)$.

Note that for $\ln(4 - 5x)$ to be a real number, we must have $4 - 5x > 0$. Thus, we must solve the inequality $4 - 5x > 0$.

$4 - 5x > 0$

$-5x > -4$  
Subtract 4 from both sides.

$x < \frac{4}{5}$  
Divide both sides by $-5$. This reverses the inequality symbol.

Step 2: Now, we return to the inequality $\ln(4 - 5x) < 3$.

$\ln(4 - 5x) < 3$

$e^{\ln(4 - 5x)} < e^3$  
Apply properties of inequality for exponential functions.

$4 - 5x < e^3$  
Simplify. ($\ln(e^x) = x$ for all $x$)

$-5x < e^3 - 4$  
Subtract 4 from both sides.

$x > \frac{4 - e^3}{5}$  
Divide both sides by $-5$. This reverses the inequality symbol.
To determine the solution set, we combine the results from steps 1 and 2 above.

First, we must have $x$ less than $\frac{4}{5}$. Secondly, we must also have $x$ greater than $\frac{4 - e^3}{5} \approx -3.22$.

Thus, the solution set in interval notation is $\left[ \frac{4 - e^3}{5}, \frac{4}{5} \right)$.

Additional Example 4:

Solve the inequality: $12 \left( \frac{3}{2} - 0.8^x \right) \leq 3$

Solution:

\[
12 \left( \frac{3}{2} - 0.8^x \right) \leq 3
\]

\[
\frac{3}{2} - 0.8^x \leq \frac{1}{4}
\]

Divide both sides by 12.

\[
-0.8^x \leq -\frac{5}{4}
\]

Subtract $\frac{3}{2}$ from both sides.

\[
0.8^x \geq \frac{5}{4}
\]

Divide both sides by $-1$. This reverses the inequality symbol.

\[
\ln \left( 0.8^x \right) \geq \ln \left( \frac{5}{4} \right)
\]

Apply properties of inequality for logarithmic functions.

\[
x \ln (0.8) \geq \ln \left( \frac{5}{4} \right)
\]

Use laws of logarithms. \( \log_a(AB) = \log_a(A) - \log_a(B) \)

\[
x \leq \frac{\ln \left( \frac{5}{4} \right)}{\ln (0.8)}
\]

Divide both sides by $\ln(0.8)$. This reverses the inequality symbol since $\ln(0.8) < 0$.

\[
x \leq \frac{\ln(5) - \ln(4)}{\ln(4) - \ln(5)}
\]

Use laws of logarithms. \( \log_a \left( \frac{A}{B} \right) = \log_a(A) - \log_a(B) \)

\[
x \leq -1
\]

Simplify.

The solution set in interval notation is $(-\infty, -1]$. 

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Solve each of the following equations.

(a) Solve for \( x \). (Note: Further simplification may occur in step (b).) Write the answer in terms of natural logarithms, unless no logarithms are involved.

(b) Rewrite the answer so that each individual logarithm from part (a) is written as a sum or difference of logarithms of prime numbers, e.g., \( \ln(40) = \ln\left(2^3 \cdot 5\right) = \ln(2^3) + \ln(5) = 3\ln(2) + \ln(5) \).

Write the answer in simplest form.

(c) If the simplified answer from part (b) contains logarithms, rewrite the answer as a decimal, correct to the nearest thousandth; otherwise, simply rewrite the answer.

1. \( 7^x = 12 \)
2. \( 8^x = 3 \)
3. \( e^x = 22 \)
4. \( e^x = 45 \)
5. \( 3 \cdot 6^x = 42 \)
6. \( 2 \cdot 9^x = 48 \)
7. \( e^x + 7 = 3 \)
8. \( e^x - 9 = -3 \)
9. \( 19^{-3x} = 7 \)
10. \( 5^{-2x} = 3 \)
11. \( 6e^x = 13 \)
12. \( 8e^x = 18 \)
13. \( e^{5x+4} - 5 = 31 \)
14. \( 2e^{7-3x} = -20 \)
15. \( 7^{x+4} = 10 \)

16. \( 8^{3x-2} = 5 \)
17. \( \frac{x}{32} = 20 \)
18. \( 12^5 = 18 \)
19. \( 5^{4x} = 6^{7x-1} \)
20. \( 7^{x+3} = 15^{8x} \)
21. \( 2^{3x-1} = 8^{2x+5} \)
22. \( 12^{4x-3} = 45^{x+7} \)
23. \( \frac{5}{3-e^{-x}} = 7 \)
24. \( \frac{3}{2-e^{-x}} = 4 \)
25. \( e^{2x} - 3e^x - 40 = 0 \)
26. \( e^{2x} + 5e^x - 14 = 0 \)
27. \( 3^{2x} + 2\left(3^x\right) - 15 = 0 \)
28. \( 5^{2x} - 4\left(5^x\right) - 21 = 0 \)
29. \( 49^x - 7^x - 6 = 0 \)
30. \( 4^x + 8 \cdot 2^x + 7 = 0 \)
31. \( x^2 \cdot 7^x - 9 \cdot 7^x = 0 \)
32. \( x^2 \cdot 2^x - 16 \cdot 2^x = 0 \)
33. \( x^2 e^x - 4xe^x - 5e^x = 0 \)
34. \( x^2 e^x - 8xe^x + 12e^x = 0 \)
35. \( 2\log_2(3x-7) = 3x - 7 \)
36. \( 6\log_6(5x+9) = 5x + 9 \)
Exercise Set 3.4: Exponential and Logarithmic Equations and Inequalities

Solve each of the following equations. Write all answers in simplest form.

37. \(\ln x = 8\)
38. \(\ln x = 12\)
39. \(\ln x = -7\)
40. \(\ln x = -10\)
41. \(\log x = 3\)
42. \(\log x = -2\)
43. \(\log_5 (x + 1) = 2\)
44. \(\log_2 (3x + 7) = 5\)
45. \(\log (4x - 1) + 3 = 5\)
46. \(\log (2x + 1) - 7 = -4\)
47. \(\ln(3x + 8) - 6 = 3\)
48. \(\ln(5x - 4) + 2 = 9\)
49. \(\ln x + \ln(x - 5) = \ln 6\)
50. \(\ln x + \ln(x - 7) = \ln 18\)
51. \(\log_2 x + \log_2 (x - 2) = 3\)
52. \(\log_6 x + \log_6 (x + 1) = 1\)
53. \(\log_2 (x + 3) + \log_2 (x + 2) = 1\)
54. \(\log_6 (x + 2) + \log_6 (x + 3) = 1\)
55. \(\log_7 (x + 8) - \log_7 (x - 5) = \log_7 4\)
56. \(\log_2 (x + 7) - \log_2 (x - 9) = \log_2 11\)
57. \(\log_5 (3x + 5) - \log_5 (x + 11) = 0\)
58. \(\log_6 (2x + 1) - \log_6 (x - 5) = 0\)
59. \(\log_2 (x + 4) = 3 + \log_2 (x + 5)\)
60. \(\log_3 (15x + 8) = 2 + \log_3 (x + 1)\)
61. \(\log 3 + \log(x + 4) = \log 5 + \log(x - 3)\)
62. \(\ln 6 + \ln(x - 8) = \ln 9 + \ln(x + 3)\)
63. \(2 \log x = \log 2 + \log(x + 12)\)
64. \(2 \log x = \log 4 + \log(x + 8)\)
65. \(\ln \left[ \ln (x) \right] = -5\)
66. \(\ln \left[ \ln (x) \right] = 5\)
67. \(e^{\ln(x)} = 5\)
68. \(e^{\ln(x)} = -5\)
69. \(\ln \left( x^4 \right) = 4 \ln (x)\)
70. \(\left[ \ln (x) \right]^4 = 4 \ln (x)\)
71. \(\log_7 \left( 3x^2 \right) = 2 \log_7 (3x)\)
72. \(\log_7 \left( 3x \right)^2 = 2 \log_7 (3x)\)
73. \(2 \log(x) = \log 25\)
74. \(2 \ln (x) = \ln (100)\)
Exercise Set 3.4: Exponential and Logarithmic Equations and Inequalities

For each of the following inequalities,

(a) Solve for \( x \). (Note: Further simplification may occur in step (b).) Write the answer in terms of natural logarithms, unless no logarithms are involved.

(b) Rewrite the answer so that each individual logarithm from part (a) is written as a sum or difference of logarithms of prime numbers, e.g., \( \ln(40) = \ln(2^3 \cdot 5) = \ln(2^3) + \ln(5) \)
\[ = 3 \ln(2) + \ln(5). \]

Simplify further if possible, and then write the answer in interval notation.

(c) If the simplified answer from part (b) contains logarithms, rewrite the answer as a decimal, correct to the nearest thousandth; otherwise, simply rewrite the answer. Then write this answer in interval notation.

75. \( 8^{x^3} > 15 \)
76. \( 7^{x^4} \leq 2 \)
77. \( 7^x > 0 \)
78. \( 7^x < 0 \)
79. \( e^{x^2-x} \leq 9 \)
80. \( e^{x+x^4} > 10 \)
81. \( 0.2^x < 7 \)
82. \( 0.8^x \geq 6 \)
83. \( 8(0.4^x - 2) > 109 \)
84. \( 9(3 - 0.75^x) \leq 11 \)
85. \( 5(8 + e^x) \leq -6 \)
86. \( (9 - e^x) \leq 4 \)
87. \( \log_2(x) \geq 0 \)
88. \( \log_2(x) < 0 \)
89. \( \ln(7 - 3x) \leq 0 \)
90. \( \ln(5x + 2) < 0 \)